Field/Circuit Coupling for the Simulation of Quenches in Superconducting Magnets

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In this paper, we propose an optimized field/circuit coupling approach for the simulation of magnetothermal transients in superconducting magnets. The approach improves the convergence of the iterative coupling scheme between a magnetothermal partial differential model and an electrical lumped-element circuit. Such a multi-physics, multi-rate and multi-scale problem requires a consistent formulation and a dedicated framework to tackle its challenging transient effects occurring at both circuit and magnet level during normal operation and in case of faults. We derive an equivalent magnet model at the circuit side for linear and non-linear settings and discuss the convergence of the overall scheme in the framework of optimized Schwarz methods. The efficiency of the developed approach is illustrated by a numerical example of an accelerator dipole magnet with accompanying protection system.

***Index Terms*—Convergence of numerical methods, coupling circuits, eddy currents, iterative methods.**

I. Introduction

Superconducting magnets produce high magnetic fields used in high-energy particle accelerators to control the trajectory of beams of particles. In order to reach the superconducting state, they are operated at very low temperatures (1.9 K) and are prone to quench, that is, become resistive. This may result in catastrophic damage in the magnet and circuit. Quench protection systems such as the coupling-loss induced quench system are affecting both a magnet and the circuit and their mutual influence has to be carefully studied. Thus, field-circuit coupling is inevitable.

In [1], various approaches of field and circuit coupling have been put into the context of waveform relaxation methods [2]. This paper interprets the coupling conditions in terms of optimized Schwarz methods, e.g. [3]. In contrast to prior works, e.g. [4], no numerical optimization is carried out. Instead a new series expansion of the operator is proposed. This allows a better understanding of existing methods and opens the door for higher-order 'transmission conditions' as known from domain decomposition. The effectiveness is demonstrated by a a simulation of CERN's quench protection system.

Starting from Maxwell's equations and assuming a magnetoquasistatic (MQS) setting, the partial differential equation

$$\nabla \times (\nu \nabla \times \vec{A}) + \nabla \times \left(\nu_\mathrm{s} \tau_\mathrm{eq} \nabla \times \frac{\partial \vec{A}}{\partial t}\right) = \vec{\chi}_\mathrm{s} \mathbf{i} \tag{1}$$

is obtained on a domain Ω. The eddy currents are taken into account in the coil domain Ω_s by an homogenization model [5], [6] using the (nonlinear) time constant τ_eq. $\vec{A}$ is the magnetic vector potential, ν and ν_s the nonlinear reluctivity and $\mathbf{i}$ the lumped currents through each coil. Finally, $\vec{\chi}_\mathrm{s}$ is the stranded-conductor winding function, such that the current density $\vec{J}_\mathrm{s} = \vec{\chi}_\mathrm{s}\mathbf{i}$. The field equation (1) is coupled to a circuit via

$$\mathbf{v} = \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{\Psi}, \text{ with } \mathbf{\Psi} = \lambda \int_\Omega \vec{\chi}_\mathrm{s} \cdot \vec{A} \,\mathrm{d}\Omega, \tag{2}$$

where λ is a symmetry factor if Ω is only representing a fraction of the full model. The temperature on Ω_s can be obtained by solving the heat balance equation.

Finally, the equations describing the behavior of the circuit can be written in an abstract form as the following system

$$\mathbf{A}\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} + \mathbf{B}(\mathbf{x}, \mathbf{R})\mathbf{x} + \mathbf{P}\mathbf{i} = \mathbf{f}(t) \tag{3}$$

$$\mathbf{P}^\top \mathbf{x} - \mathbf{v} = 0, \tag{4}$$

where $\mathbf{R}$ depends on the coil temperature. In the case of modified nodal analysis (see [7]) the degrees of freedom $\mathbf{x}$ contain node potentials and currents through branches of voltage sources and inductors.

II. Waveform Relaxation

Following [1], waveform relaxation is used to simulate the coupled problem, the field-thermal part is simulated separately from the circuit. As in [3], [4], the waveform relaxation scheme is studied from the point of view of an optimized Schwarz method. Let us rewrite the (linearized) system in frequency domain and add iteration counters for a Gauss-Seidel scheme. The field-circuit problem is given by

$$\mathbf{A}j\omega\mathbf{x}^{(k+1)} + \mathbf{B}\mathbf{x}^{(k+1)} + \mathbf{P}\mathbf{i}_\mathrm{c}^{(k+1)} = \mathbf{g}(\omega) \tag{5}$$

$$\mathbf{v}_\mathrm{c}^{(k+1)} = \mathbf{P}^\top \mathbf{x}^{(k+1)} \tag{6}$$

$$\mathbf{v}_\mathrm{c}^{(k+1)} = \mathbf{v}_\mathrm{m}^{(k)} \tag{7}$$

and the second system (MQS) representing the spatial discretization of (1-2) as e.g. obtained by Finite Elements

$$\mathbf{K}_\mathrm{s} j\omega\mathbf{a}^{(k+1)} + \mathbf{K}_\nu \mathbf{a}^{(k+1)} = \mathbf{X}\mathbf{i}_\mathrm{m}^{(k+1)} \tag{8}$$

$$\lambda \mathbf{X}^\top j\omega \mathbf{a}^{(k+1)} = \mathbf{v}_\mathrm{m}^{(k+1)} \tag{9}$$

$$\mathbf{i}_\mathrm{m}^{(k+1)} = \mathbf{i}_\mathrm{c}^{(k+1)}. \tag{10}$$

For the optimization of convergence, the first transmission condition (7) is generalized to the linear combination

$$\mathbf{v}_{\mathrm{c}}^{(k+1)} = \alpha \mathbf{i}_{\mathrm{c}}^{(k+1)} - \alpha \mathbf{i}_{\mathrm{m}}^{(k)} + \mathbf{v}_{\mathrm{m}}^{(k)}. \tag{11}$$

A contraction factor $\rho(\alpha) < 1$ can be computed, such that

$$||\mathbf{v}_{\mathrm{c}}^{(k+1)} - \mathbf{v}_{\mathrm{c}}^{(k)}|| = |\rho(\alpha)| \; ||\mathbf{v}_{\mathrm{c}}^{(k)} - \mathbf{v}_{\mathrm{c}}^{(k-1)}||.$$

Optimal convergence is attained for $\rho(\alpha) = 0$, that is

$$\alpha = \mathbf{Z}(\omega) = j\omega\lambda\mathbf{X}^{\top}(\mathbf{K}_{\mathrm{s}} j\omega + \mathbf{K}_{\nu})^{-1}\mathbf{X},$$

which is the impedance of the magnetoquasistatic system. For small frequencies, the inverse can be expanded as a Neumann series. The lowest order term can be used as an approximation

$$\mathbf{Z}(\omega) \approx j\omega\mathbf{L} := j\omega\lambda\mathbf{X}^{\top}\mathbf{K}_{\nu}^{-1}\mathbf{X}. \tag{12}$$

This leads to the optimized transmission condition

$$\mathbf{\Psi}_{\mathrm{c}}^{(k+1)} = \mathbf{L}\mathbf{i}_{\mathrm{c}}^{(k+1)} - \mathbf{L}\mathbf{i}_{\mathrm{m}}^{(k)} + \mathbf{\Psi}_{\mathrm{m}}^{(k)}, \tag{13}$$

with the magnetic flux linkage $\mathbf{\Psi}_{\star} = \frac{1}{j\omega}\mathbf{v}_{\star}$. This special case corresponds to considering the field model in the circuit as an inductance with a correction term as already proposed in [8]. For nonlinear cases, we propose a new simplified procedure: the differential inductance

$$\mathbf{L}_{\mathrm{m}} = \lambda\mathbf{X}^{\top}\left(\frac{\mathrm{d}}{\mathrm{d}\mathbf{a}}\Big(\mathbf{K}_{\nu}(\mathbf{a})\mathbf{a}\Big)\right)^{-1}\mathbf{X} \tag{14}$$

is extracted at a working point $\mathbf{a}_{\mathrm{m}}$ and is kept constant for the subsequent waveform relaxation.

The described method is illustrated with a numerical example of the single aperture dipole magnet D1 [9]. The field and thermal equations are discretized using COMSOL MULTIPHYSICS®. All parameters are defined as specified in [9]. We consider the magnet operating at 5 kA and discharged by a resistor $R_{\mathrm{EE}} = 0.1\Omega$. The circuit is modeled and simulated in ORCAD PSPICE®. The co-simulation was established with CERN's in-house coupling tool STEAM (Simulation of Transient Effects in Accelerator Magnets) [10].

To analyze the influence of the inductance estimation on the waveform relaxation convergence, the (scalar) differential inductance (14) is multiplied by a scaling coefficient k_{L}.

Fig. 1 shows the number of iterations needed to obtain a sufficiently accurate solution (error $< 10^{-3}$). For $k_{\mathrm{L}} = 0.9$ the convergence is obtained in the least number of iterations, as inter-filament coupling losses dissipate the magnetic energy in the coil due to the decrease of the effective differential inductance. On the other hand, the case of $k_{\mathrm{L}} = 0.5$ requires up to 16 iterations instead of the optimal 2 iterations if $\rho = 0$.

III. Conclusion

This paper has discussed multiphysyical field/circuit waveform relaxation for the specific eddy-current model used in quench simulation. A new polynomial series expansion has been proposed to speed up convergence. Numerical simulations of an aperture dipole magnet underline the importance of an optimization of the coupling conditions, as iterations per window could be significantly reduced. The full paper will feature a rigorous analysis of the Neumann series and more simulation results.

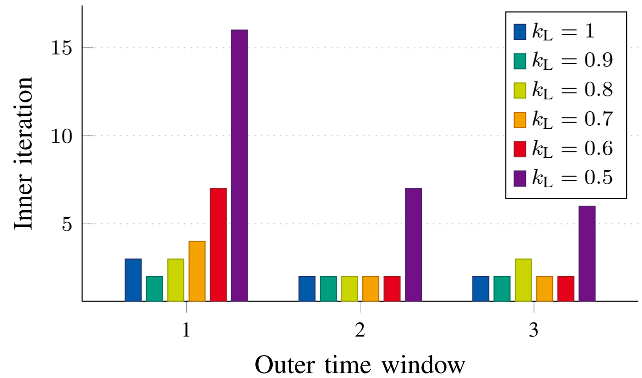

Figure 1. Convergence comparison of the first 3 time windows

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